General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC1

Unit Pure Core 1

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Express $\dfrac{5\sqrt{3}-6}{2\sqrt{3}+3}$ in the form $m + n\sqrt{3}$, where m and n are integers. *(4 marks)*

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2 The line AB has equation $4x - 3y = 7$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the straight line that is parallel to AB and which passes through the point $C(3, -5)$, giving your answer in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(b) The line AB intersects the line with equation $3x - 2y = 4$ at the point D. Find the coordinates of D. *(3 marks)*

(c) The point E with coordinates $(k-2, 2k-3)$ lies on the line AB. Find the value of the constant k. *(2 marks)*

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3 The polynomial $\text{p}(x)$ is given by

$$\text{p}(x) = x^3 + 2x^2 - 5x - 6$$

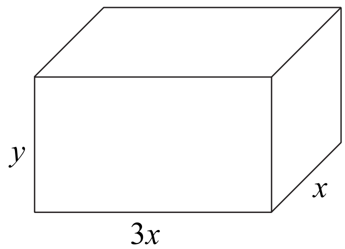
(a) (i) Use the Factor Theorem to show that $x + 1$ is a factor of $\text{p}(x)$. *(2 marks)*

(ii) Express $\text{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Verify that $\text{p}(0) > \text{p}(1)$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 + 2x^2 - 5x - 6$, indicating the values where the curve crosses the x-axis. *(3 marks)*

4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is $32\ \text{cm}^2$.

(a) (i) Show that $3x^2 + 4xy = 16$. *(2 marks)*

(ii) Hence show that the volume, $V\ \text{cm}^3$, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$ *(2 marks)*

(b) Find $\frac{dV}{dx}$. *(2 marks)*

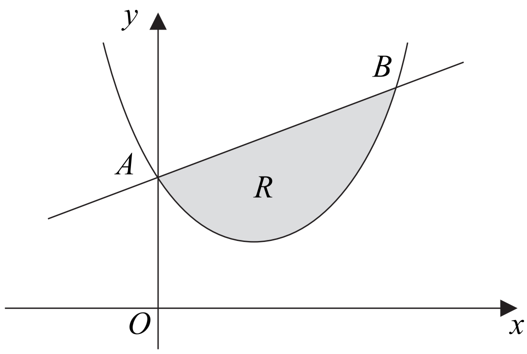
(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$. *(2 marks)*

(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$. *(2 marks)*

5 (a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

(b) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.

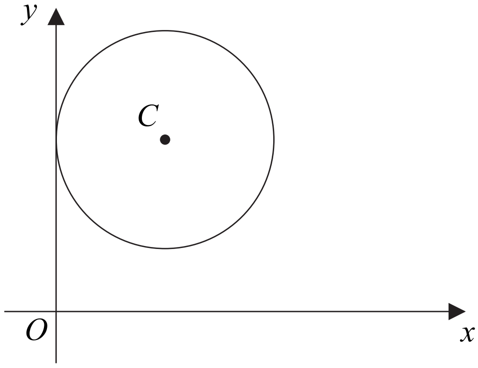


(i) Find the coordinates of the point B. *(3 marks)*

(ii) Find $\int (x^2 - 3x + 5)\,dx$. *(3 marks)*

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. *(4 marks)*

6 The circle with centre $C(5,\ 8)$ touches the y-axis, as shown in the diagram.



(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(b) (i) Verify that the point $A(2,\ 12)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the tangent to the circle at the point A, giving your answer in the form $sx + ty + u = 0$, where s, t and u are integers. *(5 marks)*

(c) The points P and Q lie on the circle, and the mid-point of PQ is $M(7,\ 12)$.

(i) Show that the length of CM is $n\sqrt{5}$, where n is an integer. *(2 marks)*

(ii) Hence find the area of triangle PCQ. *(3 marks)*

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7 The gradient, $\dfrac{dy}{dx}$, of a curve C at the point $(x,\ y)$ is given by

$$\frac{dy}{dx} = 20x - 6x^2 - 16$$

(a) (i) Show that y is increasing when $3x^2 - 10x + 8 < 0$. *(2 marks)*

(ii) Solve the inequality $3x^2 - 10x + 8 < 0$. *(4 marks)*

(b) The curve C passes through the point $P(2,\ 3)$.

(i) Verify that the tangent to the curve at P is parallel to the x-axis. *(2 marks)*

(ii) The point $Q(3,\ -1)$ also lies on the curve. The normal to the curve at Q and the tangent to the curve at P intersect at the point R. Find the coordinates of R.

(7 marks)